

Contracrostipunctus

Achilles has come to visit his friend and jogging companion, the Tortoise, at his home.

Achilles: Heavens, you certainly have an admirable boomerang collection!

Tortoise: Oh, pshaw. No better than that of any other Tortoise. And now, would you like to step into the parlor?

Achilles: Fine. (*Walks to the corner of the room.*) I see you also have a large collection of records. What sort of music do you enjoy?

Tortoise: Sebastian Bach isn't so bad, in my opinion. But these days, I must say, I am developing more and more of an interest in a rather specialized sort of music.

Achilles: Tell me, what kind of music is that?

Tortoise: A type of music which you are most unlikely to have heard of. I call it "music to break phonographs by".

Achilles: Did you say "to break phonographs by"? That is a curious concept. I can just see you, sledgehammer in hand, whacking one phonograph after another to pieces, to the strains of Beethoven's heroic masterpiece *Wellington's Victory*.

Tortoise: That's not quite what this music is about. However, you might find its true nature just as intriguing. Perhaps I should give you a brief description of it?

Achilles: Exactly what I was thinking.

Tortoise: Relatively few people are acquainted with it. It all began when my friend the Crab—have you met him, by the way?—paid me a visit.

Achilles: 'twould be a pleasure to make his acquaintance, I'm sure. Though I've heard so much about him, I've never met him.

Tortoise: Sooner or later I'll get the two of you together. You'd hit it off splendidly. Perhaps we could meet at random in the park one day . . .

Achilles: Capital suggestion! I'll be looking forward to it. But you were going to tell me about your weird "music to smash phonographs by", weren't you?

Tortoise: Oh, yes. Well, you see, the Crab came over to visit one day. You must understand that he's always had a weakness for fancy gadgets, and at that time he was quite an aficionado for, of all things, record players. He had just bought his first record player, and being somewhat gullible, believed every word the salesman had told him about it—in particular, that it was capable of reproducing any and all sounds. In short, he was convinced that it was a Perfect phonograph.

- Achilles:* Naturally, I suppose you disagreed.
- Tortoise:* True, but he would hear nothing of my arguments. He staunchly maintained that any sound whatever was reproducible on his machine. Since I couldn't convince him of the contrary, I left it at that. But not long after that, I returned the visit, taking with me a record of a song which I had myself composed. The song was called "I Cannot Be Played on Record Player 1".
- Achilles:* Rather unusual. Was it a present for the Crab?
- Tortoise:* Absolutely. I suggested that we listen to it on his new phonograph, and he was very glad to oblige me. So he put it on. But unfortunately, after only a few notes, the record player began vibrating rather severely, and then with a loud "pop", broke into a large number of fairly small pieces, scattered all about the room. The record was utterly destroyed also, needless to say.
- Achilles:* Calamitous blow for the poor fellow, I'd say. What was the matter with his record player?
- Tortoise:* Really, there was nothing the matter, nothing at all. It simply couldn't reproduce the sounds on the record which I had brought him, because they were sounds that would make it vibrate and break.
- Achilles:* Odd, isn't it? I mean, I thought it was a Perfect phonograph. That's what the salesman had told him, after all.
- Tortoise:* Surely, Achilles, you don't believe everything that salesmen tell you! Are you as naïve as the Crab was?
- Achilles:* The Crab was naïver by far! I know that salesmen are notorious prevaricators. I wasn't born yesterday!
- Tortoise:* In that case, maybe you can imagine that this particular salesman had somewhat exaggerated the quality of the Crab's piece of equipment . . . perhaps it was indeed less than Perfect, and could not reproduce every possible sound.
- Achilles:* Perhaps that is an explanation. But there's no explanation for the amazing coincidence that your record had those very sounds on it . . .
- Tortoise:* Unless they got put there deliberately. You see, before returning the Crab's visit, I went to the store where the Crab had bought his machine, and inquired as to the make. Having ascertained that, I sent off to the manufacturers for a description of its design. After receiving that by return mail, I analyzed the entire construction of the phonograph and discovered a certain set of sounds which, if they were produced anywhere in the vicinity, would set the device to shaking and eventually to falling apart.
- Achilles:* Nasty fellow! You needn't spell out for me the last details: that you recorded those sounds yourself, and offered the dastardly item as a gift . . .

- Tortoise:* Clever devil! You jumped ahead of the story! But that wasn't the end of the adventure, by any means, for the Crab did not believe that his record player was at fault. He was quite stubborn. So he went out and bought a new record player, this one even more expensive, and this time the salesman promised to give him double his money back in case the Crab found a sound which it could not reproduce exactly. So the Crab told me excitedly about his new model, and I promised to come over and see it.
- Achilles:* Tell me if I'm wrong—I bet that before you did so, you once again wrote the manufacturer, and composed and recorded a new song called "I Cannot Be Played on Record Player 2", based on the construction of the new model.
- Tortoise:* Utterly brilliant deduction, Achilles. You've quite got the spirit.
- Achilles:* So what happened this time?
- Tortoise:* As you might expect, precisely the same thing. The phonograph fell into innumerable pieces, and the record was shattered.
- Achilles:* Consequently, the Crab finally became convinced that there can be no such thing as a Perfect record player.
- Tortoise:* Rather surprisingly, that's not quite what happened. He was sure that the next model up would fill the bill, and having twice the money, he—
- Achilles:* Oho—I have an idea! He could have easily outwitted you, by obtaining a LOW-fidelity phonograph—one that was not capable of reproducing the sounds which would destroy it. In that way, he would avoid your trick.
- Tortoise:* Surely, but that would defeat the original purpose—namely, to have a phonograph which could reproduce any sound whatsoever, even its own self-breaking sound, which is of course impossible.
- Achilles:* That's true. I see the dilemma now. If any record player—say Record Player X—is sufficiently high-fidelity, then when it attempts to play the song "I Cannot Be Played on Record Player X", it will create just those vibrations which will cause it to break . . . So it fails to be Perfect. And yet, the only way to get around that trickery, namely for Record Player X to be of lower fidelity, even more directly ensures that it is not Perfect. It seems that every record player is vulnerable to one or the other of these frailties, and hence all record players are defective.
- Tortoise:* I don't see why you call them "defective". It is simply an inherent fact about record players that they can't do all that you might wish them to be able to do. But if there is a defect anywhere, it is not in THEM, but in your expectations of what they should be able to do! And the Crab was just full of such unrealistic expectations.

- Achilles:* Compassion for the Crab overwhelms me. High fidelity or low fidelity, he loses either way.
- Tortoise:* And so, our little game went on like this for a few more rounds, and eventually our friend tried to become very smart. He got wind of the principle upon which I was basing my own records, and decided to try to outfox me. He wrote to the phonograph makers, and described a device of his own invention, which they built to specification. He called it "Record Player Omega". It was considerably more sophisticated than an ordinary record player.
- Achilles:* Let me guess how: Did it have no moving parts? Or was it made of cotton? Or—
- Tortoise:* Let me tell you, instead. That will save some time. In the first place, Record Player Omega incorporated a television camera whose purpose it was to scan any record before playing it. This camera was hooked up to a small built-in computer, which would determine exactly the nature of the sounds, by looking at the groove-patterns.
- Achilles:* Yes, so far so good. But what could Record Player Omega do with this information?
- Tortoise:* By elaborate calculations, its little computer figured out what effects the sounds would have upon its phonograph. If it deduced that the sounds were such that they would cause the machine in its present configuration to break, then it did something very clever. Old Omega contained a device which could disassemble large parts of its phonograph subunit, and rebuild them in new ways, so that it could, in effect, change its own structure. If the sounds were "dangerous", a new configuration was chosen, one to which the sounds would pose no threat, and this new configuration would then be built by the rebuilding subunit, under direction of the little computer. Only after this rebuilding operation would Record Player Omega attempt to play the record.
- Achilles:* Aha! That must have spelled the end of your tricks. I bet you were a little disappointed.
- Tortoise:* Curious that you should think so . . . I don't suppose that you know Gödel's Incompleteness Theorem backwards and forwards, do you?
- Achilles:* Know WHOSE Theorem backwards and forwards? I've never heard of anything that sounds like that. I'm sure it's fascinating, but I'd rather hear more about "music to break records by". It's an amusing little story. Actually, I guess I can fill in the end. Obviously, there was no point in going on, and so you sheepishly admitted defeat, and that was that. Isn't that exactly it?
- Tortoise:* What! It's almost midnight! I'm afraid it's my bedtime. I'd love to talk some more, but really I am growing quite sleepy.

- Achilles:* As am I. Well, I'll be on my way. (*As he reaches the door, he suddenly stops, and turns around.*) Oh, how silly of me! I almost forgot, I brought you a little present. Here. (*Hands the Tortoise a small, neatly wrapped package.*)
- Tortoise:* Really, you shouldn't have! Why, thank you very much indeed. I think I'll open it now. (*Eagerly tears open the package, and inside discovers a glass goblet.*) Oh, what an exquisite goblet! Did you know that I am quite an aficionado for, of all things, glass goblets?
- Achilles:* Didn't have the foggiest. What an agreeable coincidence!
- Tortoise:* Say, if you can keep a secret, I'll let you in on something: I'm trying to find a Perfect goblet: one having no defects of any sort in its shape. Wouldn't it be something if this goblet—let's call it "G"—were the one? Tell me, where did you come across Goblet G?
- Achilles:* Sorry, but that's MY little secret. But you might like to know who its maker is.
- Tortoise:* Pray tell, who is it?
- Achilles:* Ever hear of the famous glassblower Johann Sebastian Bach? Well, he wasn't exactly famous for glassblowing—but he dabbled at the art as a hobby, though hardly a soul knows it—and this goblet is the last piece he blew.
- Tortoise:* Literally his last one? My gracious. If it truly was made by Bach, its value is inestimable. But how are you sure of its maker?
- Achilles:* Look at the inscription on the inside—do you see where the letters 'B', 'A', 'C', 'H' have been etched?
- Tortoise:* Sure enough! What an extraordinary thing. (*Gently sets Goblet G down on a shelf.*) By the way, did you know that each of the four letters in Bach's name is the name of a musical note?
- Achilles:* 't isn't possible, is it? After all, musical notes only go from 'A' through 'G'.
- Tortoise:* Just so; in most countries, that's the case. But in Germany, Bach's own homeland, the convention has always been similar, except that what we call 'B', they call 'H', and what we call 'B-flat', they call 'B'. For instance, we talk about Bach's "Mass in B Minor", whereas they talk about his "H-moll Messe". Is that clear?
- Achilles:* . . . hmm . . . I guess so. It's a little confusing: H is B, and B is B-flat. I suppose his name actually constitutes a melody, then.
- Tortoise:* Strange but true. In fact, he worked that melody subtly into one of his most elaborate musical pieces—namely, the final *Contrapunctus* in his *Art of the Fugue*. It was the last fugue Bach ever wrote. When I heard it for the first time, I had no idea how it would end. Suddenly, without warning, it broke off. And then . . . dead silence. I realized immediately that was where Bach died. It is an indescribably sad moment, and the effect it had on me was—shattering. In any case, B-A-C-H is the last theme of that fugue. It is hidden inside the piece. Bach didn't point it out

226

Musical score for measures 226-229. It consists of four staves: two treble clefs and two bass clefs. The music is in G major and 4/4 time. Measure 226 shows the beginning of a new section with a treble staff starting on a G4 and a bass staff on a G2. The music features complex rhythmic patterns and chromaticism.

230

Musical score for measures 230-233. It consists of four staves: two treble clefs and two bass clefs. The music continues with intricate counterpoint. Measure 230 features a treble staff with a series of sixteenth-note runs. The bass staff provides a steady accompaniment.

236

Musical score for measures 236-239. It consists of four staves: two treble clefs and two bass clefs. In measure 236, a box is drawn around the notes in the bass staff: G2, A2, B2, C3, D3, E3, F3, G3. Below the box, the text "(1685-1750)" is written. The music concludes with a final cadence in measure 239.

FIGURE 19. The last page of Bach's Art of the Fugue. In the original manuscript, in the handwriting of Bach's son Carl Philipp Emanuel, is written: "N.B. In the course of this fugue, at the point where the name B.A.C.H. was brought in as countersubject, the composer died." (B-A-C-H in box.) I have let this final page of Bach's last fugue serve as an epitaph. [Music printed by Donald Byrd's program "SMUT", developed at Indiana University.]

explicitly, but if you know about it, you can find it without much trouble. Ah, me—there are so many clever ways of hiding things in music . . .

Achilles: . . . or in poems. Poets used to do very similar things, you know (though it's rather out of style these days). For instance, Lewis Carroll often hid words and names in the first letters (or characters) of the successive lines in poems he wrote. Poems which conceal messages that way are called "acrostics".

Tortoise: Bach, too, occasionally wrote acrostics, which isn't surprising. After all, counterpoint and acrostics, with their levels of hidden meaning, have quite a bit in common. Most acrostics, however, have only one hidden level—but there is no reason that one couldn't make a double-decker—an acrostic on top of an acrostic. Or one could make a "contracrostic"—where the initial letters, taken in reverse order, form a message. Heavens! There's no end to the possibilities inherent in the form. Moreover, it's not limited to poets; anyone could write acrostics—even a dialogician.

Achilles: A dial-a-logician? That's a new one on me.

Tortoise: Correction: I said "dialogician", by which I meant a writer of dialogues. Hmm . . . something just occurred to me. In the unlikely event that a dialogician should write a contrapuntal acrostic in homage to J. S. Bach, do you suppose it would be more proper for him to acrostically embed his OWN name—or that of Bach? Oh, well, why worry about such frivolous matters? Anybody who wanted to write such a piece could make up his own mind. Now getting back to Bach's melodic name, did you know that the melody B-A-C-H, if played upside down and backwards, is exactly the same as the original?

Achilles: How can anything be played upside down? Backwards, I can see—you get H-C-A-B—but upside down? You must be pulling my leg.

Tortoise: 'pon my word, you're quite a skeptic, aren't you? Well, I guess I'll have to give you a demonstration. Let me just go and fetch my fiddle—*(Walks into the next room, and returns in a jiffy with an ancient-looking violin.)*—and play it for you forwards and backwards and every which way. Let's see, now . . . *(Places his copy of the Art of the Fugue on his music stand and opens it to the last page.)* . . . here's the last *Contrapunctus*, and here's the last theme . . .

The Tortoise begins to play: B-A-C- — but as he bows the final H, suddenly, without warning, a shattering sound rudely interrupts his performance. Both he and Achilles spin around, just in time to catch a glimpse of myriad fragments of glass tinkling to the floor from the shelf where Goblet G had stood, only moments before. And then . . . dead silence.

Consistency, Completeness, and Geometry

Implicit and Explicit Meaning

IN CHAPTER II, we saw how meaning—at least in the relatively simple context of formal systems—arises when there is an isomorphism between rule-governed symbols, and things in the real world. The more complex the isomorphism, in general, the more “equipment”—both hardware and software—is required to extract the meaning from the symbols. If an isomorphism is very simple (or very familiar), we are tempted to say that the meaning which it allows us to see is explicit. We see the meaning without seeing the isomorphism. The most blatant example is human language, where people often attribute meaning to words in themselves, without being in the slightest aware of the very complex “isomorphism” that imbues them with meanings. This is an easy enough error to make. It attributes all the meaning to the *object* (the word), rather than to the *link* between that object and the real world. You might compare it to the naïve belief that noise is a necessary side effect of any collision of two objects. This is a false belief; if two objects collide in a vacuum, there will be no noise at all. Here again, the error stems from attributing the noise exclusively to the *collision*, and not recognizing the role of the *medium*, which carries it from the objects to the ear.

Above, I used the word “isomorphism” in quotes to indicate that it must be taken with a grain of salt. The symbolic processes which underlie the understanding of human language are so much more complex than the symbolic processes in typical formal systems, that, if we want to continue thinking of meaning as mediated by isomorphisms, we shall have to adopt a far more flexible conception of what isomorphisms can be than we have up till now. In my opinion, in fact, the key element in answering the question “What is consciousness?” will be the unraveling of the nature of the “isomorphism” which underlies meaning.

Explicit Meaning of the *Contracrostipunctus*

All this is by way of preparation for a discussion of the *Contracrostipunctus*—a study in levels of meaning. The Dialogue has both explicit and implicit meanings. Its most explicit meaning is simply the story

which was related. This “explicit” meaning is, strictly speaking, extremely *implicit*, in the sense that the brain processes required to understand the events in the story, given only the black marks on paper, are incredibly complex. Nevertheless, we shall consider the events in the story to be the explicit meaning of the Dialogue, and assume that every reader of English uses more or less the same “isomorphism” in sucking that meaning from the marks on the paper.

Even so, I’d like to be a little more explicit about the explicit meaning of the story. First I’ll talk about the record players and the records. The main point is that there are two levels of meaning for the grooves in the records. Level One is that of music. Now what is “music”—a sequence of vibrations in the air, or a succession of emotional responses in a brain? It is both. But before there can be emotional responses, there have to be vibrations. Now the vibrations get “pulled” out of the grooves by a record player, a relatively straightforward device; in fact you can do it with a pin, just pulling it down the grooves. After this stage, the ear converts the vibrations into firings of auditory neurons in the brain. Then ensue a number of stages in the brain, which gradually transform the linear sequence of vibrations into a complex pattern of interacting emotional responses—far too complex for us to go into here, much though I would like to. Let us therefore content ourselves with thinking of the sounds in the air as the “Level One” meaning of the grooves.

What is the Level Two meaning of the grooves? It is the sequence of vibrations induced in the record player. This meaning can only arise after the Level One meaning has been pulled out of the grooves, since the vibrations in the air cause the vibrations in the phonograph. Therefore, the Level Two meaning depends upon a chain of *two* isomorphisms:

- (1) isomorphism between arbitrary groove patterns and air vibrations;
- (2) isomorphism between arbitrary air vibrations and phonograph vibrations.

This chain of two isomorphisms is depicted in Figure 20. Notice that isomorphism 1 is the one which gives rise to the Level One meaning. The Level Two meaning is more implicit than the Level One meaning, because it is mediated by the chain of two isomorphisms. It is the Level Two meaning which “backfires”, causing the record player to break apart. What is of interest is that the production of the Level One meaning forces the production of the Level Two meaning simultaneously—there is no way to have Level One without Level Two. So it was the implicit meaning of the record which turned back on it, and destroyed it.

Similar comments apply to the goblet. One difference is that the mapping from letters of the alphabet to musical notes is one more level of isomorphism, which we could call “transcription”. That is followed by “translation”—conversion of musical notes into musical sounds. Thereafter, the vibrations act back on the goblet just as they did on the escalating series of phonographs.

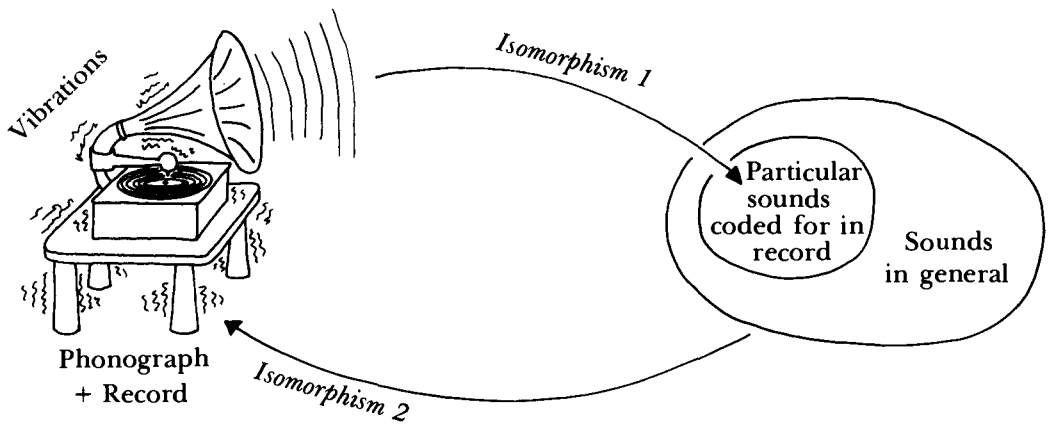


FIGURE 20. Visual rendition of the principle underlying Gödel's Theorem: two back-to-back mappings which have an unexpected boomeranging effect. The first is from groove-patterns to sounds, carried out by a phonograph. The second—familiar, but usually ignored—is from sounds to vibrations of the phonograph. Note that the second mapping exists independently of the first one, for any sound in the vicinity, not just ones produced by the phonograph itself, will cause such vibrations. The paraphrase of Gödel's Theorem says that for any record player, there are records which it cannot play because they will cause its indirect self-destruction. [Drawing by the author.]

Implicit Meanings of the *Contracrostipunctus*

What about implicit meanings of the Dialogue? (Yes, it has more than one of these.) The simplest of these has already been pointed out in the paragraphs above—namely, that the events in the two halves of the dialogue are roughly isomorphic to each other: the phonograph becomes a violin, the Tortoise becomes Achilles, the Crab becomes the Tortoise, the grooves become the etched autograph, etc. Once you notice this simple isomorphism, you can go a little further. Observe that in the first half of the story, the Tortoise is the perpetrator of all the mischief, while in the second half, he is the victim. What do you know, but his own method has turned around and backfired on him! Reminiscent of the backfiring of the records' music—or the goblet's inscription—or perhaps of the Tortoise's boomerang collection? Yes, indeed. The story is about backfiring on two levels, as follows . . .

Level One: Goblets and records which backfire;

Level Two: The Tortoise's devilish method of exploiting implicit meaning to cause backfires—which backfires.

Therefore we can even make an isomorphism between the two levels of the story, in which we equate the way in which the records and goblet boomerang back to destroy themselves, with the way in which the Tortoise's own fiendish method boomerangs back to get him in the end. Seen this

way, the story itself is an example of the backfirings which it discusses. So we can think of the *Contracrostipunctus* as referring to itself indirectly, in that its own structure is isomorphic to the events it portrays. (Exactly as the goblet and records refer implicitly to themselves via the back-to-back isomorphisms of playing and vibration-causing.) One may read the Dialogue without perceiving this fact, of course—but it is there all the time.

Mapping Between the *Contracrostipunctus* and Gödel's Theorem

Now you may feel a little dizzy—but the best is yet to come. (Actually, some levels of implicit meaning will not even be discussed here—they will be left for you to ferret out.) The deepest reason for writing this Dialogue was to illustrate Gödel's Theorem, which, as I said in the Introduction, relies heavily on two different levels of meaning of statements of number theory. Each of the two halves of the *Contracrostipunctus* is an “isomorphic copy” of Gödel's Theorem. Because this mapping is the central idea of the Dialogue, and is rather elaborate, I have carefully charted it out below.

phonograph	\Leftrightarrow	axiomatic system for number theory
low-fidelity phonograph	\Leftrightarrow	“weak” axiomatic system
high-fidelity phonograph	\Leftrightarrow	“strong” axiomatic system
“Perfect” phonograph	\Leftrightarrow	complete system for number theory
“blueprint” of phonograph	\Leftrightarrow	axioms and rules of formal system
record	\Leftrightarrow	string of the formal system
playable record	\Leftrightarrow	theorem of the axiomatic system
unplayable record	\Leftrightarrow	nontheorem of the axiomatic system
sound	\Leftrightarrow	true statement of number theory
reproducible sound	\Leftrightarrow	interpreted theorem of the system
unreproducible sound	\Leftrightarrow	true statement which isn't a theorem
song title:	\Leftrightarrow	implicit meaning of Gödel's string:
“I Cannot Be Played on Record Player X”		“I Cannot Be Derived in Formal System X”

This is not the full extent of the isomorphism between Gödel's Theorem and the *Contracrostipunctus*, but it is the core of it. You need not worry if you don't fully grasp Gödel's Theorem by now—there are still a few Chapters to go before we reach it! Nevertheless, having read this Dialogue, you have already tasted some of the flavor of Gödel's Theorem without necessarily being aware of it. I now leave you to look for any other types of implicit meaning in the *Contracrostipunctus*. “Quaerendo invenietis!”